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SPECTRAL CHARACTERISTICS OF TWO-DIMENSIONAL TURBULENT CONVECTION IN A VERTICAL SLOT

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The spatial spectra of two-dimensional turbulent convection are obtained in [1]: the velocity fluctuation energy in a developed turbulent convective flow follows the law $E(k) \sim k^{-11/5}$ while the temperature fluctuation energy follows $E_T(k) \sim k^{-7/5}$. The possibility of realizing turbulent flow with such spectral dependences in a vertical slot with heat insulated boundaries is shown there. The energy distribution over the spectrum depends substantially on the heat elimination conditions on the slot side walls. Flow in a slot with ideally heat conductive walls is examined in this paper. The exponential realization of plane turbulent flow in a Hele–Shaw convective cell heated from below which is formed by plates conducting heat well with a linear temperature distribution along the height is described.

1. Incompressible viscous fluid flow is considered in a plane vertical layer of thickness d with the characteristic dimension $\ell \gg d$ (Fig. 1), with boundaries of infinite heat conductivity and the vertical temperature gradient $\partial T/\partial y = -a$. The motion is considered planar ($\mathbf{v} \gg (v_x, v_y, 0)$) with a given velocity profile and temperature across the layer

$$\mathbf{v} = \mathbf{v}(x, y, t)\sin(\pi z/d),\ T = -ay + \Theta(x, y, t)\sin(\pi z/d). \qquad (1.1)$$

Substitution of (1.1) in the equation of thermogravitational convection in the Boussinesq approximation [2] with subsequent integration with respect to z between 0 and d results in two-dimensional equations which take the form after being made dimensionless

$$\partial\mathbf{v}/\partial t = -(\pi/4)(\mathbf{v}\nabla)\mathbf{v} - \nabla p + \Delta\mathbf{v} - D\mathbf{v} + \xi\mathrm{Gr}(\Theta - y); \qquad (1.2)$$

$$\partial\Theta/\partial t = -(\pi/4)(\mathbf{v}\nabla)\Theta + (\Delta\Theta - D\Theta)/\mathrm{Pr} + \mathbf{v}\xi; \qquad (1.3)$$

$$\nabla\mathbf{v} = 0. \qquad (1.4)$$

Here $\mathrm{Pr} = \nu/\chi$ is the Prandtl number; $\mathrm{Gr} = g\beta l^4 a/\nu^2$, Grashoff number; ν, viscosity; χ, thermal diffusivity; β, coefficient of thermal expansion; ξ, a unit vector along the y axis; $D = \pi^2 l^2/d^2$, friction (viscous in (1.2) and thermal in (1.3)) on the side walls of the cavity. Selected as units for measuring the length, time, velocity, and temperature are ℓ, ℓ^2/ν, ν/ℓ, $a\ell$.

The spectral characteristics and investigated on the basis of a hierarchical model of turbulent convection constructed in [1] by projecting the equations of motion (1.2) on a special basis describing the hierarchy of the vortices and thermics of progressively diminishing scale

$$\mathbf{v} = \sum_{N,n} A_{Nn}(t)\,\mathbf{v}_{Nn}(x, y),\ \Theta = \sum_{N,n} C_{Nn}(t)\,\Theta_{Nn}(x, y).$$

A singularity of the basis functions is the fact that the functions with different subscript N corresponding to the vortex dimension have Fourier transforms that do not overlap in the

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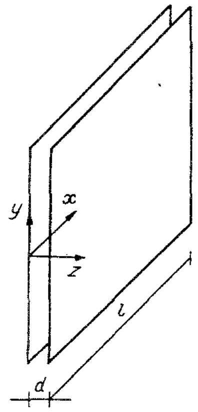

Fig. 1

wave vector space. An increase in N by one corresponds to a 2 times diminution in the vortex dimensions. The subscript n governs the location of a specific vortex of a given scale.

To obtain a small-parameter model of the Galerkin equation for the coefficients A_{Nn} and C_{Nn} and averaged with respect to the subscript n. A system is obtained for A_N and C_N and satisfying the role of collective variables corresponding to velocity and temperature field perturbations whose wave vectors lie within one octave:

$$\dot{A}_N = \sum_{M,L} T_{NML} A_M A_L + (K_N - D) A_N + \mathrm{Gr}\, F_N C_N; \quad (1.5)$$

$$\dot{C}_N = \sum_{M,L} H_{NML} A_M C_L + (K_N - D) C_N/\mathrm{Pr} + P_N A_N, \quad (1.6)$$

where $K_N = -21.4 \cdot 2^{2N}$; $F_N = 3.3 \cdot 2^N$; $P_N = 0.15 \cdot 2^{-N}$; $T_{NML} = 2^N T_{0,M-N,L-N}$; $H_{NML} = 2^N H_{0,M-N,L-N}$. Tables of values of the elements of the matrices T_{0ML} and H_{0ML} as well as the whole elucidation of the derivation of the model equations and the matrix calculation can be found in [1].

The motion being considered is one of the few possibilities for obtaining a two-dimensional turbulent flow under laboratory conditions. However, the method of creating a turbulent convective motion imposes a substantial imprint on the nature of the process, including even on the energy distribution over the spectrum. The energy influx is described by the terms $\mathrm{Gr}F_N C_N$ in (1.5) and $P_N A_N$ in (1.6). Here the energy pumping is realized in the whole motion scale. The influence of viscosity is also modified. The main energy dissipation occurs because of friction on the side walls, which is identically effective for motions of any scale (the terms DA_N and DC_N in (1.5) and (1.6), respectively).

If similarity modes are set up in isothermal turbulence because of the balance of nonlinear terms describing the distribution of energy between vortices of different scales, then in the case under consideration the similarity mode can be assumed only by a balance of terms describing energy pumping in motion of a given scale with dissipative and nonlinear terms.

We seek the power laws for A_N and C_N in the form $A_N = A_0 2^{-\varkappa N}$, $C_N = C_0 2^{-\lambda N}$. For the convective terms to cancel the friction on the side walls it is necessary that $\mathrm{Gr}F_N C_N \sim DA_N$ and $P_N A_N \sim DC_N/\mathrm{Pr}$, which is possible only for $\lambda = \varkappa + 1$. The balance between the energy influx from the vortices of other scales and its dissipation requires $T_{NML} A_M A_L \sim DA_N$ and $H_{NML} A_M C_L \sim DC_N$. This is valid for $\varkappa = 1$ independently of the energy distribution of the temperature fluctuations. The balance of the nonlinear terms in (1.5) with the convective $T_{NML} A_M A_L \sim \mathrm{Gr} F_N C_N$ is possible for $\lambda = 2\varkappa$ while the equilibrium of the nonlinear terms in (1.6) with the term describing the generation of thermal perturbations by velocity vortices that exist in the background of linear temperature distribution $H_{NML} A_M C_L \sim P_N A_N$, requires $\lambda = 2$. All the relationships presented for $\varkappa$ and λ are satisfied for $\varkappa = 1$ and $\lambda = 2$, i.e., $A_N = A_0 2^{-N}$, $C_N = C_0 2^{-2N}$. Such a distribution of the vortex amplitudes corresponds to the "-3" law for both the velocity fluctuation spectrum and the temperature fluctuation spectrum: $E(k) \sim k^{-3}$, $E_T(k) \sim k^{-3}$.

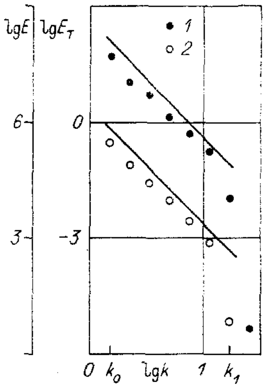

Fig. 2

The velocity fluctuation spectrum agrees with the "−3" spectrum for isothermal two-dimensional turbulence that is set up in an internal entropy transfer interval in which the energy flux in the spectrum is zero. The nature of the "−3" interval in the case under consideration is different but the velocity fluctuation energy flux over the spectrum due to the interaction of vortex triples nevertheless equals zero. This means that the energy of the moving fluid dissipates in the same scales as generates but a constant flux of entropy exists from the large- to the small-scale motion.

Presented in Fig. 2 are results of solving the system (1.5) and (1.6) for values of the parameters corresponding to the experimental model described above: $Gr = 1.4 \cdot 10^{10}$, $Pr = 0.7$, $D = 55{,}000$. Points 1 and 2 are values of E and E_T and k_0 and k_1 are the minimal and maximal wave numbers representing the plane motion in the cavity.

2. Air flow in a cavity of 1200 × 1200 × 16 mm size was investigated experimentally. The cavity dimensions were selected so as to assure developed turbulent flow in the slot plane while conserving plane-parallel flow with a simple profile across the layer. The solution of the fluid stability problem in a plane vertical layer of thickness d with boundaries of infinite heat conductivity and a vertical temperature gradient yields the spectrum of critical Rayleigh numbers for flows with different velocity profiles across the layer [2] $Ra_m = (\pi^2 m^2/4 + k^2)^2$ $(m = 1, 2, \ldots)$, where Ra is the Rayleigh number defined over the layer halfwidth, k is the perturbation wave number along the x axis, the subscript m corresponds to a solution with a dependence on the coordinate z of the form $\sin(\pi m z/d)$. For the lowest mode ($m = 1$) and an infinite perturbation wavelength $Ra_1 = 6.09$, while for perturbations with a wavelength equal to twice the layer thickness $Ra_1 = 24, 36$. The minimal value of Ra_2 ($k = 0$) equals 97.4.

For the layer thickness selected and the superposed temperature difference 60°C characteristic for performing the tests, Ra = 18 calculated over the layer halfwidth, which permits computation of the realization of a plane flow with profile of the form $\sin(\pi z/d)$, while Ra found with respect to the linear dimension of the cavity and governing the nature of the two-dimensional convective flows equals $1.1 \cdot 10^{10}$ for given parameters.

A model is fabricated from four duraluminum plates of 8 mm thickness arranged in parallel. Between the inner plates is the working cavity in which the air motion is investigated. The outer plates perform the role of heat shields. The same linear temperature distribution is produced in them as in the main plates while the space between them and the inner plates is filled with glass to prevent convection. The temperature on the cavity faces was maintained with 0.1°C accuracy.

All the measurements refer to the temperature field and were executed by 64 thermocouples mounted with an uniform spacing along the horizontal and the central sections of the cavity and connected to the thermocouple commutator with 40-Hz operating frequency. Control of the measurement process, the processing and the delivery of the results were realized by using an IVK based on an SM-4 electronic counter.

The results of measuring a one-dimensional spatial temperature fluctuation spectrum, obtained by averaging 300 realizations taken with a 1-min interval and noted by dots in Fig. 3. The line corresponds to a k^{-3} diminution law. If the power-law dependences in the energetic $E(k)$ and one-dimensional $E_1(k)$ spectra agree in homogeneous isotropic turbulence, then

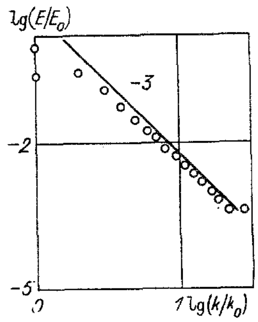

Fig. 3

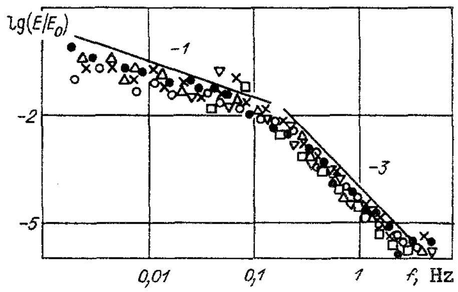

Fig. 4

for measurements in a bounded domain the intensity of the perturbations with wavelength on the order of the domain dimensions in a one-dimensional spectrum drops since there is no contribution of more large-scale perturbations because of the masking effect [3].

Represented in Fig. 4 are results of measuring the time spectrum of the temperature fluctuations at different points of the cavity. From 5 to 20 realizations were taken off 1024 points from the selected thermocouples with a 0.08-1.6 sec sampling period. The spectrum averaged over the series of measurements was delivered to a printer and plotter. The results obtained from one thermocouple are marked on the figure with identical symbols.

Two intervals were extracted in the spectrum. The low-frequency part of the spectrum has a law of diminution of the order $E_T(\omega) \sim \omega^{-1}$ and is subject to the law $E_T(\omega) \sim \omega^{-3}$ from the frequency ~0.1 Hz. Agreement of the powers in the high-frequency part of the tune spectrum with the power of diminution of the spatial spectrum indicates satisfaction of the Taylor hypothesis for these scales, fine-scale vortices ane carried over to the large-scale flow and the time spectrum agrees at a point with the space spectrum.

The space–time spectra of the temperature have also investigated. To this end, the lowest harmonics were extracted from the spatial spectrum, their realizations were inscribed and subjected to a Fourier time analysis. A study performed earlier of the space–time spectra of different closed convective flows showed the presence of a series of isolated independent frequencies in both the supercritical and the developed turbulent flows [4]. The flow under investigation turned out to be an exception in this plan. Measurements of the space–time spectra in time segments to 3000 sec yielded monotonically decreasing spectra with the growth of frequency without isolated frequencies.

It is interesting to note that the hierarchical model that permits obtaining space–time spectra with series of isolated frequencies [1] in the case of a vertical layer with heat insulated boundaries yields stable stationary solutions for the spatial spectra in the case considered.

Thus, the spatial spectra with quite definite inertial intervals obtained in experiment confirm the possibility of realizing plane developed turbulent flow in a thin vertical slot. At the same time, turbulence in a Hele–Shaw cell with heat-conducting boundaries tends to a strong dependence on the detailed construction of the energy-containing vortices, and respectively, on the forcing actions resulting in their formation, the small slope of the cavity, the features of the planform geometry, the temperature boundary condition on the narrow faces, etc., because of the sharp decreases in the spectral energy as the wave number ($\sim k^{-3}$) increases.

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ATOMIZATION OF A TURBULENT LAYER OF A MIXTURE*

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This article studies the problem of the atomization of a turbulent layer of a mixture formed at the interface of two incompressible media with constant but different densities. It is found that the solution tends toward similarity for long periods of time. The degree of similarity, meanwhile, cannot be determined from dimensional analysis. Instead, it is found during the solution of a boundary-value problem. The degree of similarity is a function of the empirical constants of the model. Similarity solutions are constructed for several parameters, and the dependence of the degree of similarity on the constants of the model is graphed. A formula for the degree of similarity is obtained in an approximation in which the turbulent velocity is constant with respect to the space variable, which the solution for the density of the mixture is expressed through a probability integral. A special case of problem for a uniform medium was examined in [1, 2]. The results of calculations reported there agree with the values obtained in the present study.

1. Formulation of the Problem. A space is filled with two incompressible fluids with the densities ρ_1^0 and ρ_2^0. The interface passes over a plane. Let a plane turbulent layer of the width L_0, consisting of a mixture of both substances, be created at the initial moment of time in the neighborhood of the interface. Such a state can arise, for example, due to the accelerated motion of an interface in the time interval to with the appropriate sign of acceleration. Here, a turbulent layer of the mixture of the width L_0 is created during the time t_0 and is associated with a certain initial turbulent velocity $v(x, t_0)$. In the absence of turbulence sources, the initial layer of the mixture expands and envelops adjacent fluids. The turbulent energy, determined through the characteristic turbulent velocity, decays in this case and dissipates into heat.

We will use the semiempirical model in [3] to describe the resultant turbulent mixing. This model is based on the balance equation for the kinetic turbulent energy $v^2/2$ and contains three constants. The equations are obtained from the conservation laws for a compressible fluid by means of the substitution $\rho = \bar{\rho} + \rho', u = \bar{u} + u', \ p = \bar{p} + p'$ and corresponding averaging, with the third correlations and the products of the second correlations being discarded. We find from the equation of continuity that $\partial\bar{\rho}/\partial t + \partial\bar{\rho}\tilde{u}/\partial x = 0, \ \tilde{u} = \overline{\rho' u'}/\bar{\rho}$. Here, we used the incompressibility condition $\bar{u} = 0$.

The equation for the kinetic turbulent energy follows from the continuity law and the momentum conservation law [3, 4]: $(1/2)(\partial\bar{\rho}v^2/\partial t + \tilde{u}\partial\bar{\rho}v^2/\partial x) = -\bar{\nu}\rho v^3/l + (5/6)\bar{\rho}v^2\partial\tilde{u}/\partial x$. Applying the Prandtl hypothesis $\overline{\rho' u'} = -lv\partial\bar{\rho}/\partial x$, to the equations, we have

$$\partial\rho/\partial t = \partial(lv\partial\rho/\partial x)/\partial x; \tag{1.1}$$

$$\frac{\partial\rho v^2}{2\partial t} - \frac{lv}{2}\frac{\partial \ln\rho}{\partial x}\frac{\partial\rho v^2}{\partial x} = -\frac{\nu\rho v^3}{l} + \beta\frac{\partial}{\partial x}\left(\rho lv\frac{\partial v^2}{\partial x}\right) + \frac{5\rho v^2}{6}\left[\frac{\partial\ln\rho}{\partial t} - lv\left(\frac{\partial\ln\rho}{\partial x}\right)^2\right], \tag{1.2}$$

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